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Weibull Analysis

Application to Reliability Engineering

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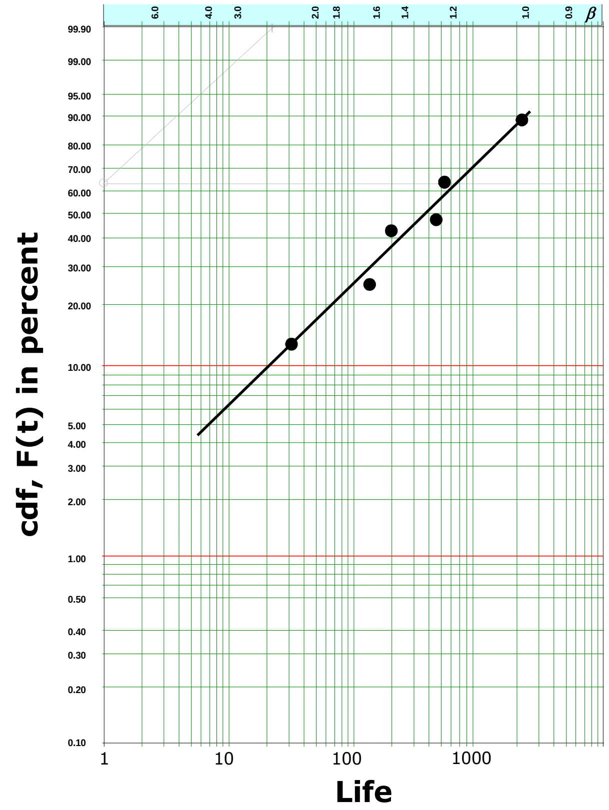

About Shashank Kotwal

- BE – KJSCE, Mumbai, MBA – SPJIMR, Mumbai
- 23 years experience in Mahindra, Crompton Greaves and Mukand
- Founder and Principal Associate – Shashank Kotwal & Associates
- Expertise – Strategy, Business Excellence, Product Development and Statistics

Email: shashank@sskotwal.in

https://www.linkedin.com/in/shashank-kotwal-3646112/

https://twitter.com/kotwal_shashank

About Us !

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Process driven business results

Strategy Formulation & Deployment

Visioning, Strategy Formulation, Cascading to Policy Deployment & BSCs, Business Planning

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Business Excellence

Facilitating the excellence journey with reference to Deming/ EFQM framework, Industry good practices, Quality engineering tools & techniques for problem solving

Product & Business Development

Product Planning, Business Case preparation for new product/projects, Strategic tie-ups, Mergers & Acquisition, Business Development

Introduction

- History
- Applications

Reliability

- Quality, Durability, Reliability
- Reliability & Life Testing

Applications

- Life Testing
- Comparison of 2 designs
- Warranty Predictions

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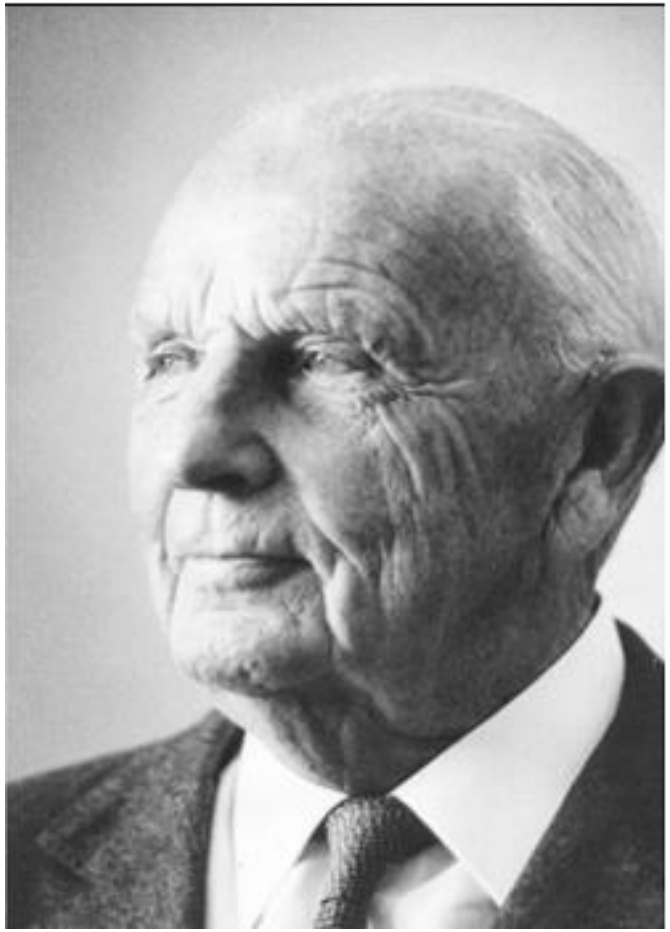
Waloddi Weibull



18 June 1887 – 12 October 1979

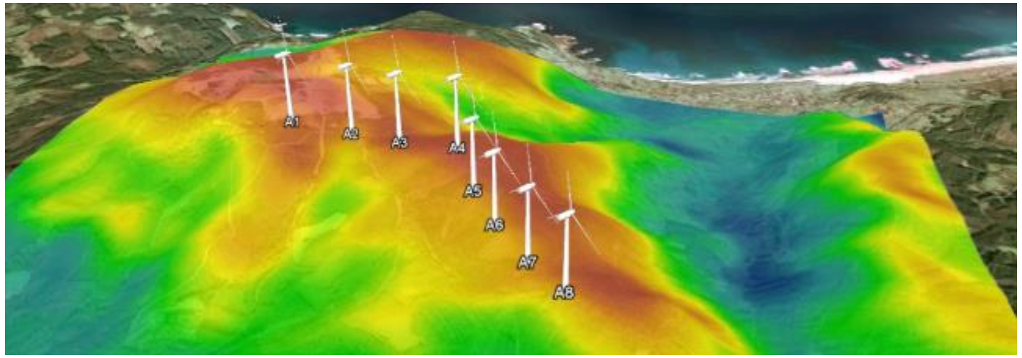
- Swedish engineer, scientist, and mathematician
- Published paper on Weibull probability distribution in 1939
- Used Weibull distribution to analyse fatigue test results of materials, rupture in solids and bearings.

Source: Wikipedia

Major Applications

RELIABILITY ESTIMATION

WIND ENERGY ASSESSMENT



Weibull Analysis for Reliability Engineering

- Weibull analysis is the process of discovering the trends in product or system failure and using them to predict future failures in similar situations.
- The primary advantage is that it can provide reasonably accurate analyses and failure forecasts with extremely small data samples.

What to expect from Weibull Analysis?

- What type of failure mechanism is the root cause?
- How many failures are expected?
- How reliable is the existing part compared to a possible new design?
- When should we replace an existing part with a new one to minimise maintenance costs?

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Prof. David Garvin's 8 dimensions of Quality

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Performance

This refers to the primary operating characteristics of a product. This is the main reason for which the Customer buys the product or service.

Conformance

Conformance is delivering what is promised in Specifications, Literature or Advertisement.

Features

Features are "Bells & Whistles" of products, those secondary characteristics that supplement the product's basic functioning.

Aesthetics

This is the most subjective dimension of quality. It is how a product looks (shape, colour& décor) ,feels, sounds & tastes or smells.

Durability

Durability is the useful life of the product

Reliability

Reliability is the probability that a part, device, equipment, or system will perform its intended function for a specified period of time under given conditions.

Serviceability

It is an ease with which the product can be repaired.

Perceived Quality

This dimension deals with the image of the manufacturer of the product or the provider of the service.

Durability

- It is the useful life of the product
- Durability is built in to the product at design stage
- It is the time period for which the customer expects the product to perform satisfactorily.
- Units of measurement:
  - Months, Years in Service
  - Number of duty cycles

Reliability

› Reliability is the probability of a product performing its intended function for a specified life under the operating conditions encountered in a manner that meets or exceeds customer expectations.

› Reliability is:

  – Focused on the probability of maintaining intended function over time.

  – Measured as a percentage.

Durability and Reliability

- Durability is the useful of the product
- Reliability is probability that it will serve the customer for the useful life.
- Reliability is always expressed as a function of time. – R(t)
- Example:
  - If a machine is claimed to have a useful life of 7 years (Durability)
  - Then, Reliability is the probability of any machine of the same design functioning satisfactorily for 7 years.

Reliability Metrics

› Reliability is reported on the basis of measurement of unreliability. Common measures of unreliability are:

- % Failure (percentage of failures in a total population)
- MTBF (Mean time between Failures)
- MTTF (Mean time to failure)
- R/1000 (Repairs per thousand)
- Bq (the life at which q% of population will fail).

B_{10} Life

- In life data analysis, the term B_{10} life is the age or time at which 10% of the population is expected to fail (90% reliability).
- Weibull originated the term believed to come from the German word Brucheinzeleitet, time-to-initial fracture.
- B or L can be used in this context which is life corresponding to a particular probability of occurrence, i.e., B_1 and L_1 both mean the age to 1% failure of the population.
- Bearings are frequently rated for a B_{10} life under specific loading conditions as recommended by Waloddi Weibull.

Life Data Analysis

Also known as Weibull Analysis

Life Data Analysis - Steps

- Gather life data for the product.
- Select a lifetime distribution that will fit the data and model the life of the product.
- Estimate the parameters that will fit the distribution to the data.
- Generate plots and results that estimate the life characteristics, like reliability or mean life, of the product.

Life Data

- The term life data refers to measurements of the life of products.
- Product lifetimes can be measured in hours, miles, cycles or any other metric that applies to the period of successful operation of a particular product.
- Since time is a common measure of life, life data points are often called "times-to-failure"

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Type of Life Data

- With **complete data**, the exact time-to-failure for the unit is known (e.g. the unit failed at 100 hours of operation).
- With **suspended or right censored data**, the unit operated successfully for a known period of time and then continued (or could have continued) to operate for an additional unknown period of time (e.g. the unit was still operating at 100 hours of operation).
- With **interval and left censored data**, the exact-time-to failure is unknown but it falls within a known time range. For example, the unit failed between 100 hours and 150 hours (interval censored) or between 0 hours and 100 hours (left censored).

Suspended Life Data

- An item is said to be suspended when it is removed from the test before failure.
  - This is also known as censoring and the data is called as censored data.
- Suspended item analysis is used when,
  - There are items in the sample that have not yet failed.
  - More items are placed on test than are expected to fail during the allotted test time.
  - It is required to make an analysis before test completion.
  - Some units may be malfunctioning, but it is unclear whether they have failed. These units are withdrawn and checked. If no failures are found, the items are suspended.

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Sample size requirement for Life Testing

- **For component level life testing**:
  - At least six samples should be tested to failure, under a given failure mode(s), and six or more 'time-to-failures' should be recorded.
  - If there are more than one predominant failure modes, then six or more samples should be tested to failure for each failure mode, and Weibull Analysis should be done for the set of data corresponding to every failure mode
- For **system level life testing**, sample size is arrived at taking into account testing time available and resources

Life Data Distributions

- Statistical distributions have been formulated by statisticians, mathematicians and engineers to mathematically model or represent certain behaviour.
- Some distributions, tend to better represent life data and are commonly called lifetime distributions or life distributions. Examples:
  - Exponential Distribution
  - Log-Normal Distribution
  - Weibull Distribution

Parameter estimation methods

- Plotting on Probability plot papers
- Regression (can be done using MS-Excel)
- Maximum likelihood estimation (MLE)

Output from Life Data Analysis

- Reliability at given time
- Probability of failure at given time
- Mean Life
- Failure Rate
- Warranty Time
- B(X) Life

Advantages of Weibull Analysis

- Is based on non-parametric statistics and is capable of assisting engineers in determining a likely distribution of failure data.
- Provides quick analysis of reliability data.
- **Permits predictions, even with small sample size (>= 6).**
- Provides associated predictive risks in terms of confidence bands.
- Allows use of suspended (censored) tests to improve reliability estimates.
  - some failures are required. Test to bogey alone is not sufficient to do the analysis.

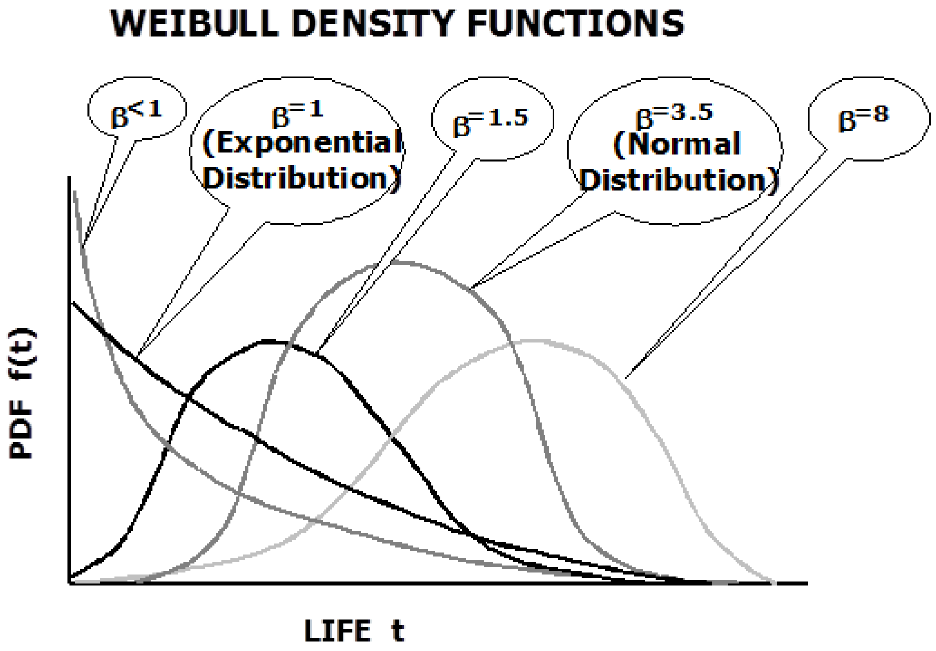
Weibull Probability Distribution

- The Cumulative Density Function (F(t)) is:

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

- The parameter β is known as the "shape parameter" or Weibull slope.
- The parameter θ is known as the "scale parameter" or characteristic life. θ is the point in time that corresponds to 63.2% cumulative failure.

Weibull Probability Density Function



- The shape of the Weibull distribution can vary widely, depending on a parameter β.
  - When β=1, the Weibull distribution is identical to the exponential distribution (constant hazard rate).
  - When β=3.5, the Weibull distribution approximates the properties of the normal distribution.

Weibull Plots

- The Weibull cumulative distribution can be represented graphically as a straight line when plotted on Weibull graph paper.
- Weibull graph paper has been designed to simplify interpretation of results by using a specially arranged scales for 'Life' and 'Percent Failed' to achieve a straight line characteristic.

WEIBULL PLOTS

Derivation

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

$$1 - F(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

$$\frac{1}{1 - F(t)} = e^{\left(\frac{t}{\theta}\right)^{\beta}}$$

$$\ln\left[\frac{1}{1 - F(t)}\right] = \left[\frac{t}{\theta}\right]^{\beta}$$

$$\ln \ln\left[\frac{1}{1 - F(t)}\right] = \beta \times \ln[t] - \beta \times \ln[\theta]$$

The above expression is similar to a straight line equation, Y=m X + C , where,

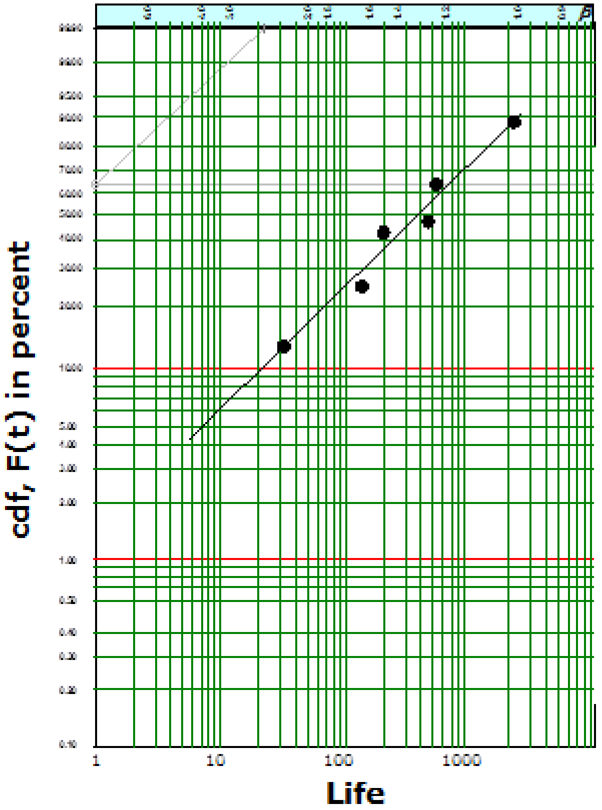
$Y = \ln \ln\left[\frac{1}{1 - F(t)}\right]$,

m = β

X = ln(t), and c = -β x ln (θ)

WEIBULL PLOT PAPER

Structure



WEIBULL PLOTS

Rewrite cdf equation:

$$1 - F(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

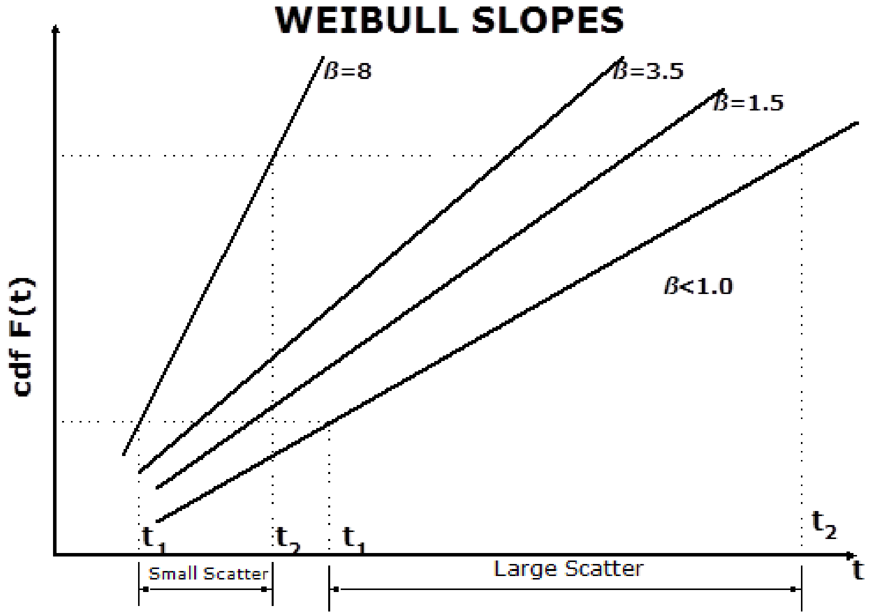
Take natural log twice to obtain,

$$\ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right) = \beta \ln(t) - \beta \ln(\theta)$$

This is a straight line of the form:

$$Y = mX + c$$

Inferences from Weibull slopes (β)



Weibull plots for various slopes on Weibull Probability Paper

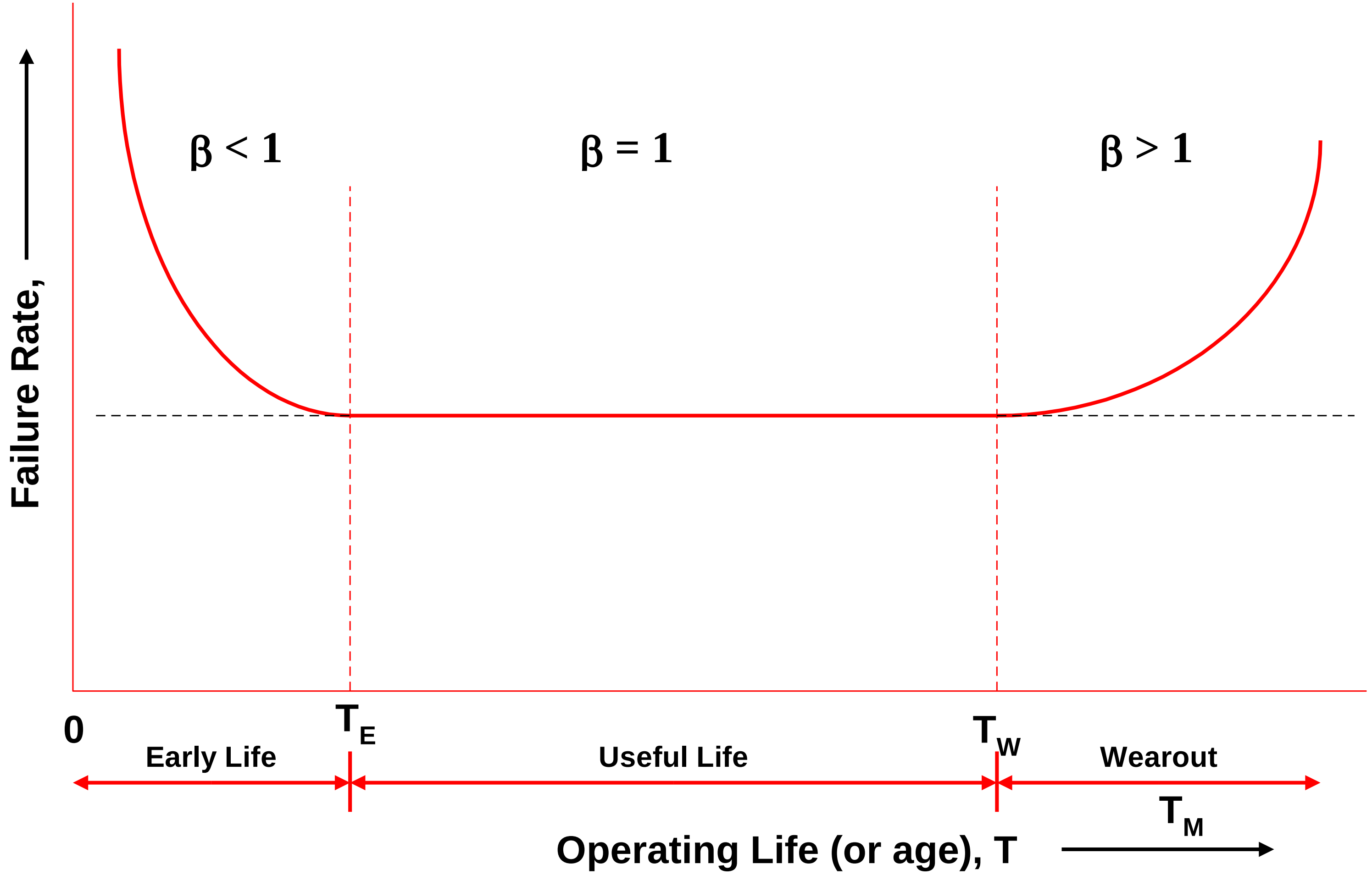
- The higher the value of slope, the lower the scatter.
- Data with low scatter indicates relatively low variation in failure times.
- Data with high scatter indicates relatively high variation in failure times

Weibull hazard function $h(t) = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{\beta - 1}$

This equation is linked to the three phases of the bathtub curve.

- When $\beta < 1$, this represents a decreasing hazard rate is the infant mortality phase.
- When $\beta = 1$, this represents a constant failure rate, the useful life phase, or random chance of failure.
- When $\beta > 1$, this represents an increasing hazard rate or wear out phase.

Mortality Curve and Hazard rate

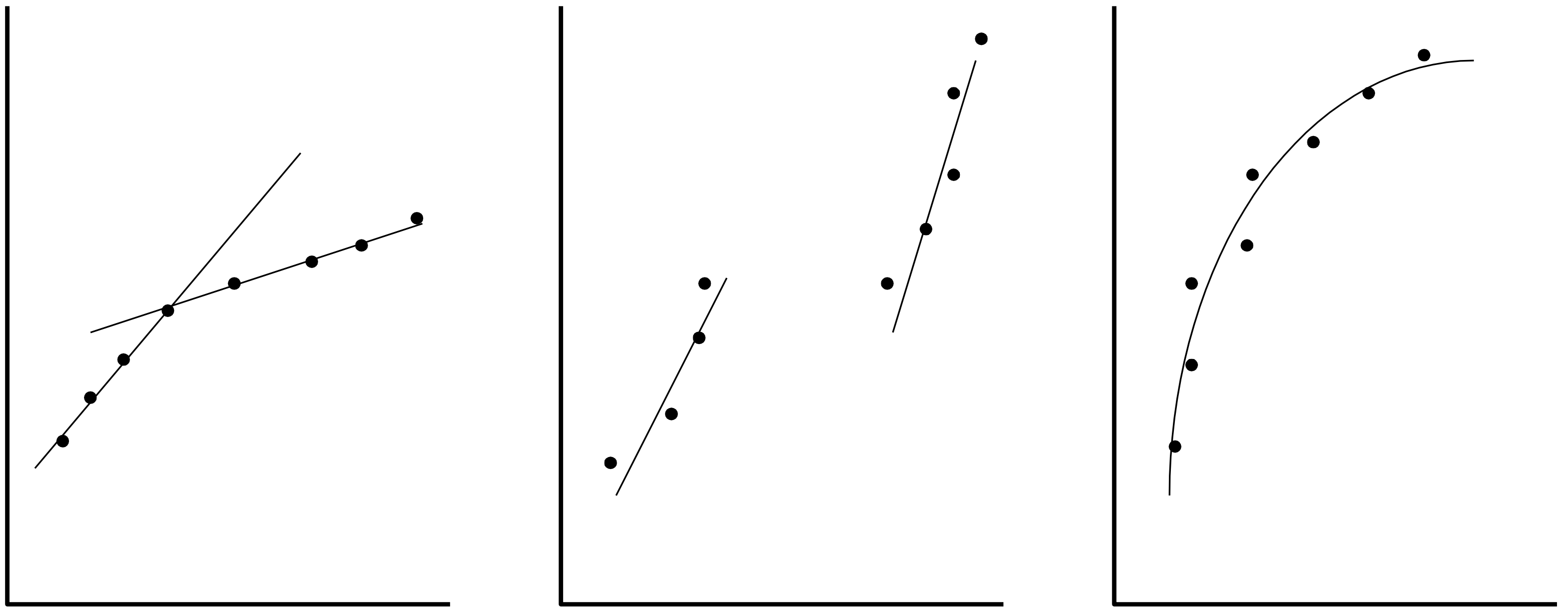


Inferences from Weibull shape parameter β

- Normally the value of β may lie in the range, $0.5 < \beta < 5$.
- If the regression analysis gives a β value outside this range, then the data integrity needs to be checked, i.e., check for data manipulation, failure recording, test completion, etc.

Inferences from Weibull plots

Non-Linearity in Weibull Plots



Mixed Failure Modes **Mixed Population** **Non-zero Min. Life**

Non-linearity – suggested next steps

- Use failure mode analysis to detect mixture of failure modes.
- Trace or know background data on test hardware to detect mixtures of populations.
- Analysis can be performed for each failure mode by considering the items failing due to other failure modes as suspended items.
- Mixed populations should be plotted separately.

Confidence bounds

- There is uncertainty in the results of Life Data Analysis, due to the limited sample sizes.
- Confidence bounds (also called confidence intervals) are used to quantify this uncertainty due to sampling error by expressing the confidence that a specific interval contains the quantity of interest, simply speaking 'the actual life may lie between the two limits'.
- Confidence bounds can be expressed as two-sided or one-sided.

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- **Comparison of 2 designs**
- **Warranty Predictions**

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Life Testing

Steps

- Rank data in increasing order (shortest to longest life).
- Assign the respective median ranks, against each failure time.
  - For each failure time, the median rank gives a typical population percentage represented by that observation.
- Plot data on Weibull paper Failure time (X) versus rank (Y)
- Determine the slope β', of the best fitting line.
- Determine the characteristic life, 'θ',
- Draw the confidence bounds (Appendix)
- Estimate the Reliability at given time (T)

CASE STUDY

A random sample of ten precision grinder wheel failures was obtained over several months of production. When a wheel fails, the number of pieces cut is recorded (in thousands). It has been assumed that the sample is representative of production, and all failures occur due to the same failure mode.

- Pieces cut at failure (cycles)
  - Wheel A: 1.2×10^5
  - Wheel B: 7.2×10^4
  - Wheel C: 1.6×10^5
  - Wheel D: 2.3×10^5
  - Wheel E: 5.4×10^4
  - Wheel F: 1.6×10^4
  - Wheel G: 9.2×10^4
  - Wheel H: 5.8×10^3
  - Wheel I: 2.1×10^4
  - Wheel J: 3.8×10^4

STEP 1

The data is ranked in increasing order.

Order No. (j)	Time to Failure
1 (wheel 'H')	5.8×10^3
2 (wheel 'F')	1.6×10^4
3 (wheel 'I')	2.1×10^4
4 (wheel 'J')	3.8×10^4
5 (wheel 'E')	5.4×10^4
6 (wheel 'B')	7.2×10^4
7 (wheel 'G')	9.2×10^4
8 (wheel 'A')	1.2×10^5
9 (wheel 'C')	1.6×10^5
10 (wheel 'D')	2.3×10^5
n=10	

STEP 2

Against each failure time, the median ranks are assigned. Median ranks are computed by the formula:

$$[(j - 0.3) / (n + 0.4)] \times 100$$

where,

j = the order number, and n = total number of samples.

Order No. (j)	Time to Failure	Median Rank (%)
1	5.8×10^3	6.7
2	1.6×10^4	16.3
3	2.1×10^4	25.9
4	3.8×10^4	35.6
5	5.4×10^4	45.2
6	7.2×10^4	54.8
7	9.2×10^4	64.4
8	1.2×10^5	74.1
9	1.6×10^5	83.7
10	2.3×10^5	93.3

STEP 3

A 3-cycle Weibull plot paper is selected.

The rational for selecting a 3-cycle Weibull paper is explained here.

Origin	1×10^3	
Minimum value	5.8×10^3	
	1×10^4	1st cycle
	1×10^5	2nd cycle
Maximum value	2.3×10^5	
	1×10^6	3rd cycle

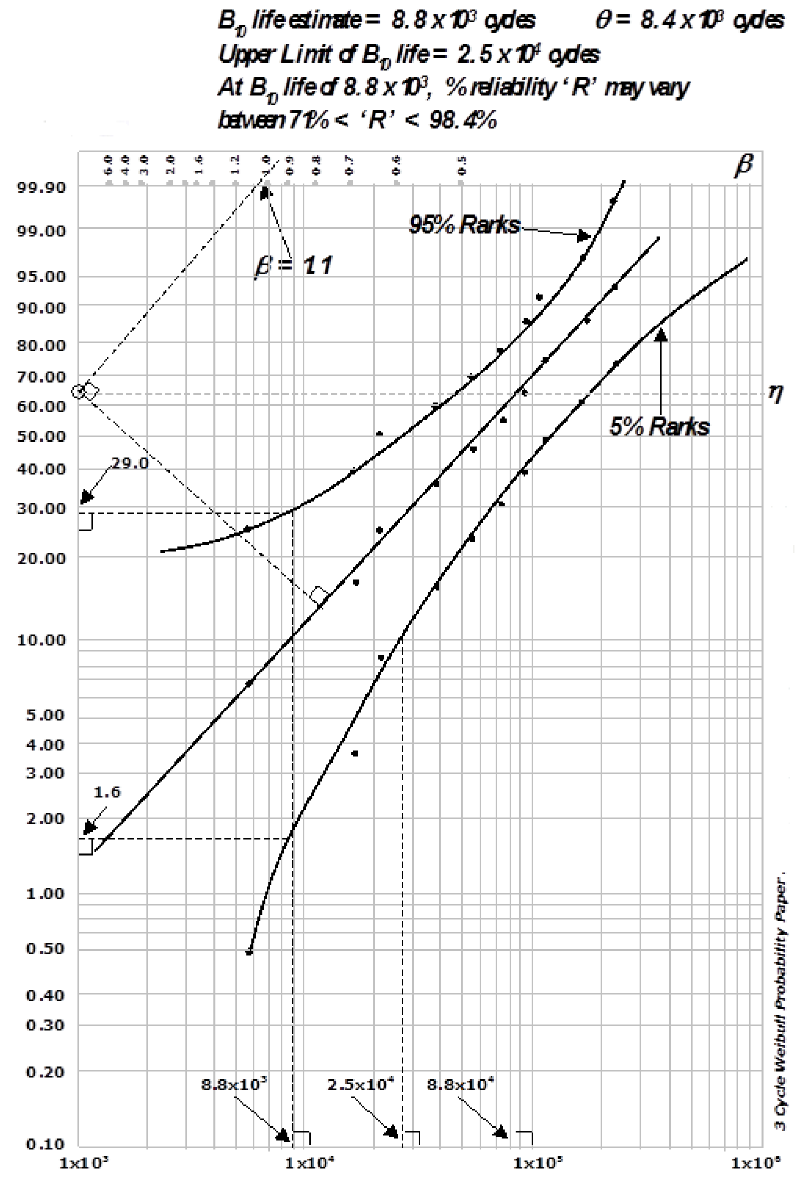
STEP 4

- The data is plotted on the Weibull plot paper.
- The slope of the best-fit line or the shape parameter β is estimated as $\beta = 1.1$.
- The characteristic life θ is estimated to be $\theta = 8.4 \times 10^4$.
- The B_{10} life is estimated to be 8.8×10^3.
- Against each failure time, the 95% and 5% ranks are assigned.
- A smooth curve connecting points in each of the sets of ranks is drawn

Order No. (j)	Time to Failure	Median Rank (%)	5% Ranks	95% Ranks
1	5.8×10^3	6.7	.5	25.89
2	1.6×10^4	16.3	3.7	39.4
3	2.1×10^4	25.9	8.7	50.7
4	3.8×10^4	35.6	15	60.8
5	5.4×10^4	45.2	22.2	69.5
6	7.2×10^4	54.8	30.4	77.8
7	9.2×10^4	64.4	39.3	87.0
8	1.2×10^5	74.1	49.3	91.3
9	1.6×10^5	83.7	60.6	96.3
10	2.3×10^5	93.3	74.1	99.5

INFERENCE

- The B_{10} life is estimated at 8.8 x 10^3 cycles and can max be up to 2.5 x 10^4 cycles
- The lower life limit has not been estimated as the curve of 5 percent rank does not intersect the 10% failure line.
- At a life of 8.8 x 10^3, the % reliability can vary from 71% (min.) to 98.4% (max.)



USING MS-EXCEL

- The same analysis can also be done using MS-Excel
- Steps up to computation of median ranks are same as shown earlier
- The next steps are shown here

› Compute the natural log of the each failure time.

› For every median rank computed, further compute the expression $\ln\left(\ln\left(\frac{1}{1 - \text{Median Rank}}\right)\right)$

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› Do regression analysis with 'log (failure time)' on X-axis, and 'ln (ln (1/(1-Median Rank)))' on the Y-axis.

› Draw a linear trend line through the regressed points.

› Estimate the 'linear regression' equation.

› This equation is the linearised form of Weibull cdf. $\ln (\ln (1/(1 - F(t)))) = \beta^* \ln (t) - \beta^* \ln (\theta)$

› The slope of the equation estimated (shape parameter, β) of the Weibull cdf.

› The scale parameter, θ of the Weibull cdf can be estimated as follows:

›
$$- \beta * \ln(\theta) = c$$
$$\therefore \theta = \exp\left(\frac{c}{\beta}\right)$$

› where,

› c = the constant estimated from the 'linear regression equation'

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USING MS-EXCEL

- Estimating B10 life

› For estimating the B10 life, substitute F(t) = 0.1 (10%).

$$\ln\left(\ln\left(\frac{1}{1-0.1}\right)\right) = \beta * \ln(t) - c$$

› The equation then reduces to:

$$-2.25 = \beta * \ln(t) - c$$

› The B10 life is then computed as:

$$t = B_{10} = \exp\left(\frac{-2.25 + c}{\beta}\right)$$

Note: *There is a method of estimating the R90C90 confidence bound using MS-Excel. It is not covered here with an aim of keeping the subject matter simple to understand. Refer Appendix*

Life Testing using suspended data

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Steps

- The data (both sets - failed and non-failed) is ranked in increasing order (shortest to longest life). Failed and suspended are marked respectively
- Next to normal order number a column of reverse order is prepared.
- For every Failed item data point, a 'Adjusted Rank' is computed using the following formula:

$$\text{Adjusted Rank (j)} = \frac{(Reverse\ Rank * Previous\ Adjusted\ Rank) + N + 1}{Reverse\ Rank + 1}$$

- Where N= the sample size (inclusive of suspended items)
- For every Adjusted Rank computed, the median rank is computed by the formula:

$$Median\ Rank = \frac{(Adjusted\ Rank(j) - 0.3)}{N + 0.4}$$

- Only the points of failures are plotted.
- Follow the steps mentioned earlier for life estimation
- **Note**: At least six failure points are necessary even when using suspended data

CASE STUDY

- This is a case study from a press-shop with 11 dies producing fender extension for automobile application.
- The output from six dies was not as desired; the reason being that the dies had been worn. The operating hours (in hundreds) for these six dies was recorded,
- The production was discontinued for change-over to another part, and the remaining five dies were replaced by other ones. These dies had clocked the certain hours of operation (in hundreds).
- The objective of this analysis is to estimate the B10 life of the dies producing fenders, using both sets of data,.

› Number of hours (00s) clocked by 6 dies which were taken out:

– 13.2, 67.8, 79.0 , 59.0, 30.0, 26.7.

› Number of hours (00s) clocked by 5 dies which were working:

– 58.0, 13.0, 75.3, 62.8, 49.5

STEP 1

- The data is arranged in the ascending order.
- For visual identification, suspended items are marked (S), and failed data points are marked in bold letters.
- The median ranks are assigned against each order.

Order Number	Failure Time	F/S	Reverse Order Number
1	13.0 (S)	S	11
2	**13.2**	**F**	**10**
3	**26.7**	**F**	**9**
4	**30.0**	**F**	**8**
5	49.5 (S)	S	7
6	58.0 (S)	S	6
7	**59.0**	**F**	**5**
8	62.8 (S)	S	4
9	**67.8**	**F**	**3**
10	75.3 (S)	S	2
11	**79.0**	**F**	**1**

STEP 2

Adjusted ranks are computed for the failed items.

E.G for Reverse order #10:

$$\frac{(Reverse\ Rank * Previous\ Adj\ Rank) + N + 1}{Reverse\ Rank + 1}$$

$$\frac{(10*0)+11+1}{10=1}=1.09$$

E.G for Reverse order #5

$$\frac{(5*3.27)+11+1}{5+1}=4.73$$

Order No	Time	F/S	Reverse Order	Adjusted rank (i)
1	13	S	11	
2	13.2	F	10	1.09
3	26.7	F	9	2.18
4	30	F	8	3.27
5	49.5	S	7	
6	58	S	6	
7	59	F	5	4.73
8	62.8	S	4	
9	67.8	F	3	6.55
10	75.3	S	2	
11	79	F	1	9.27

STEP 3

Median Ranks are computed for the Adjusted Ranks corresponding to the Failed items

Order No	Time	F/S	Reverse Order	Adjusted rank (i)	Median Ranks
1	13	S	11		
2	13.2	F	10	1.09	6.9%
3	26.7	F	9	2.18	16.5%
4	30	F	8	3.27	26.1%
5	49.5	S	7		
6	58	S	6		
7	59	F	5	4.73	38.8%
8	62.8	S	4		
9	67.8	F	3	6.55	54.8%
10	75.3	S	2		
11	79	F	1	9.27	78.7%

STEP 4

Using the Manual Method or MS-Excel the Weibull plot is made

- Inference from the Weibull plot of the suspended data

Beta, β	Theta, θ	B_{10}
1.545	7430 hours	1731 hours

Comparison of 2 designs

Steps

- Conduct the Life data analysis of 2 designs separately and estimate their respective Weibull parameters
- Compute the ratio of the mean life of two designs.
  - The higher mean life of the two designs is taken in the numerator, thus, the ratio is always greater than 1. This ratio is termed as 'Mean Life Ratio' (experimental), MLR_{Exp}.
- Compute the degrees of freedom (D.O.F.) [D.O.F. = (n1-1)*(n2-1)]
  - where, n1 and n2 are the sample sizes of design 1 and design 2 respectively.
- For the given values of beta 'β' and the calculated D.O.F., using the curves for 'test for significant difference in mean lives' (Appendix), look-up the theoretical value of Mean Life Ratio, MLR_{theo}.
- Look-up MLR_{theo} for the curves corresponding to all the three confidence levels, viz., 90%, 95% and 99%.

Steps (contd.)

- If the two designs have two different values of slope, β1 & β2, then the theoretical mean life ratio is referred for both, β1 & β2, and the average of two theoretical mean life ratios is computed.
- Compare all the MLR_{theo} with MLR_{exp},
  - if $MLR_{Exp} > MLR_{Theo}$, then it can be inferred that a significant difference in mean lives of design1 & design2 exists (at the corresponding confidence level).
  - if $MLR_{Exp} < MLR_{Theo}$, then it can be inferred that there is no significant difference in mean lives of design1 & design2 (at the corresponding confidence level).
- In case of difference in life of designs, better design (life) is decided on the Mean Life (higher the better).

CASE STUDY

Two designs of temperature sensors were put to test, to assess their life till failure. The objective is to compare two designs on the basis of the 'life-data' available

From the tests, details of life data for design ‘A’ (6 failures) and design ‘B’ (12 failures) is as shown here.

Design A (hrs)	Design B (hrs)	
105	18	950
422	314	900
446	67	1129
660	320	908
413	635	797
515	126	247

STEP 1

Following the steps (as described in the 1st & 2nd case studies) of estimating the shape parameter β and the scale parameter θ, the cumulative distribution function (cdf) for both the designs is estimated.

- The estimation results are as listed below.

Design	Shape parameter, β	Scale parameter, θ	B_{10} Life
A	1.4667	528 hours	114 hours
B	0.8658	609 hours	45 hours

STEP 2

- For both the designs the 'percentage population failed at mean' is referred from the curve of ‘β versus Percent failed at mean’
- The estimated value of 'Percent Failed at mean' is substituted in the respective CDF of the two designs and the 'Mean Life' is estimated.

Design	Shape parameter, β	% Failed at mean	Mean Life
A	1.4667	57.5%	474 hours
B	0.8658	63.5%	614 hours

STEP 3

- The experimental 'Mean Life Ratio' (MLR_{exp}), is computed.
- Since the mean-life of design **B** is greater, its value is taken in the numerator.
- Next, the degrees of freedom are calculated.

Design	Sample size (n)	Degrees of Freedom (D.O.F.) = n-1	Mean Life	MLR_{Exp} = 614/474 = 1.293
A	6	5	474	
B	12	11	614	
Combined D.O.F. = 11*5 = 55				

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STEP4

For the computed combined 'Degrees of Freedom' and the respective values of β, the 'Theoretical Mean Life Ratio', MLR_{theo} is referred at different confidence level.

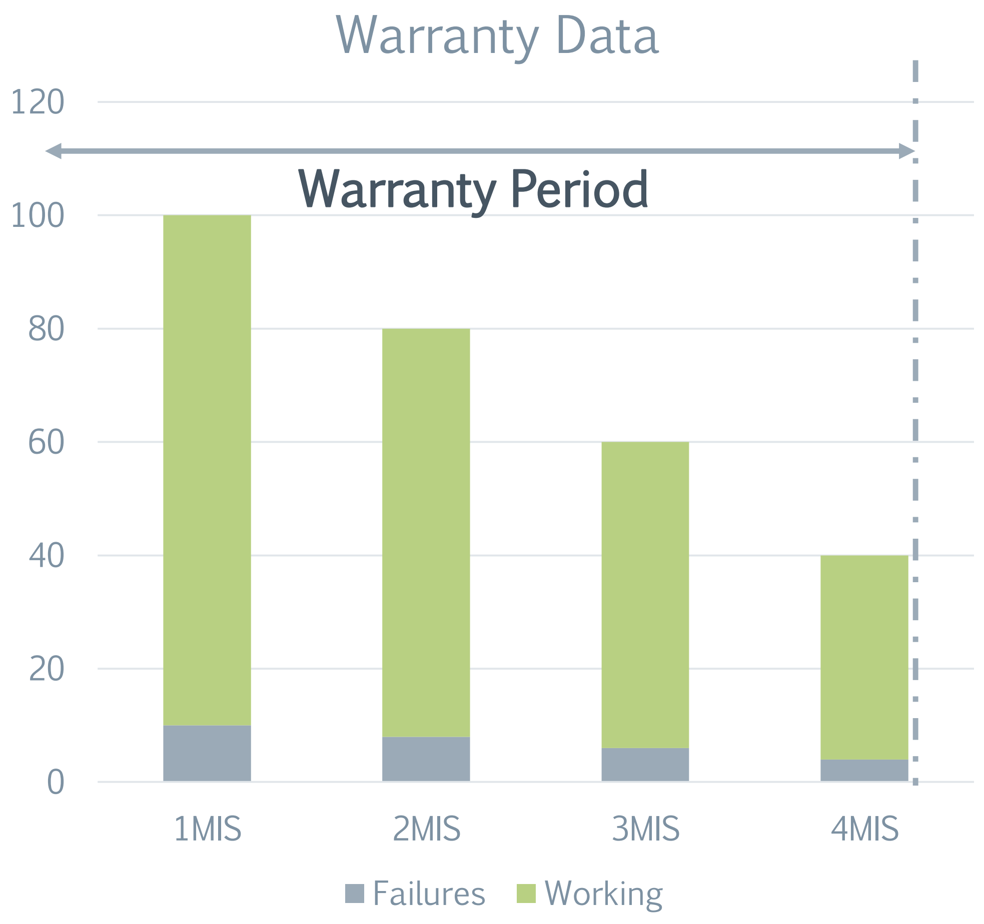
D.O.F. (1)	Confidence Level (2)	β_A = 0.8658 (3)	β_B = 1.4667 (4)	MLR_{theo} = Average of (3) & (4)
55	99	3.2	2.2	2.7
55	95	2.8	1.8	2.3
55	90	1.82	1.58	1.7

As, $MLR_{Exp} < MLR_{Theo}$, i.e., (1.2) < 2.7 or 2.3 or 1.7, it can be inferred that, based on the available life-data ***there is no significant difference in the mean life of two designs of the sensors*** and both, ***design 'A' & design 'B' have same (or comparable) average lives.***

Warranty prediction

Peculiarities of Warranty data

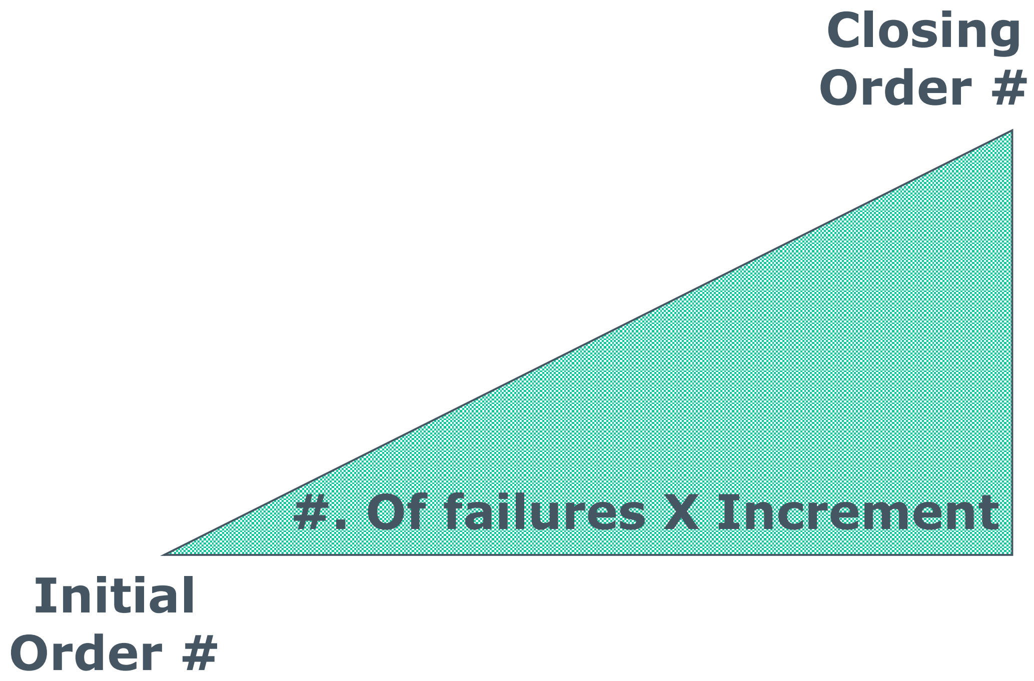
- The population of a product in field is of different ages.
- For a 'r' number of failures @ X MIS, there are 'n-r' units working properly.
- The failures are tracked only up to warranty period.



Peculiarities of Warranty data (contd.)

- There are r failures in the first period.
- But, there exact times are not known.
- Arranging the failures in ascending order is difficult.
- The order numbers are calculated by a different formula

Concept of Increment



$$\text{Increment} = \frac{(n + 1 - \text{PreviousOrder\#})}{1 + \text{Preceeding Suspended Data}}$$

n = Total population in the field

$$\text{Median Rank} = \frac{\text{Closing Order \# - 0.3}}{\text{Total Population} + 0.4}$$

CASE STUDY

A manufacturer puts out 10,000 television sets, with a 1-year warranty on the picture tube. On the basis of field data accumulated during the first 4 months, predict the total number of failed tubes during the warranty period of 1 year.

Time in service (months)	Number of units having corresponding time in service	Number of failures	Number of suspensions
0 to 1	3894	30	3864
1 to 2	2340	20	2320
2 to 3	1255	14	1241
3 to 4	1108	11	1097
	1403		1403*
Total	**10000**	**75**	**9925**

STEP

The above data are arranged in the following manner

Months in service	# of units at the MIS	Cumulative up to MIS	Failures
1	3894	3894	30
2	2340	6234	20
3	1255	7489	14
4	1108	8597	11
> 4	1403	10000	

STEP

The 'number of items following the present suspended set' are computed and arranged in a column.

Months in service	# of units at the MIS	Cumulative up to MIS	Failures	*
1	3894	3894	30	6136
2	2340	6234	20	3786
3	1255	7489	14	2525
4	1108	8597	11	1414
	1403	10000		

STEP

The new increment, order number, and median rank for every MIS is calculated.

- For the first failed set (30 failures), the new increment is calculated as follows:

$$\text{New Increment} = \frac{(N+1) - \text{Previous Order Number}}{1 + \begin{pmatrix}\text{No. of Items following present} \\ \text{suspended set}\end{pmatrix}}$$

$$\text{New Increment} = \frac{(10000+1) - 0}{1 + (6136)} = 1.63$$

- The mean order number for the thirtieth failure in the first month in service is calculated:
- Mean order number = Previous order number + [(# failures @ MIS)* new increment]
- Mean order number = 0 + [30 * 1.63] = 48.9
- There are no failures prior to 1 MIS. Hence, the previous order number is taken as zero.
- The median rank of the thirtieth failure is calculated:

$$\text{Median Rank} = \frac{\text{Order \#} - 0.3}{N + 0.4}$$

$$\text{Median Rank} = \frac{48.9 - 0.3}{10000 + 0.4} = 0.00486 = 0.486\%$$

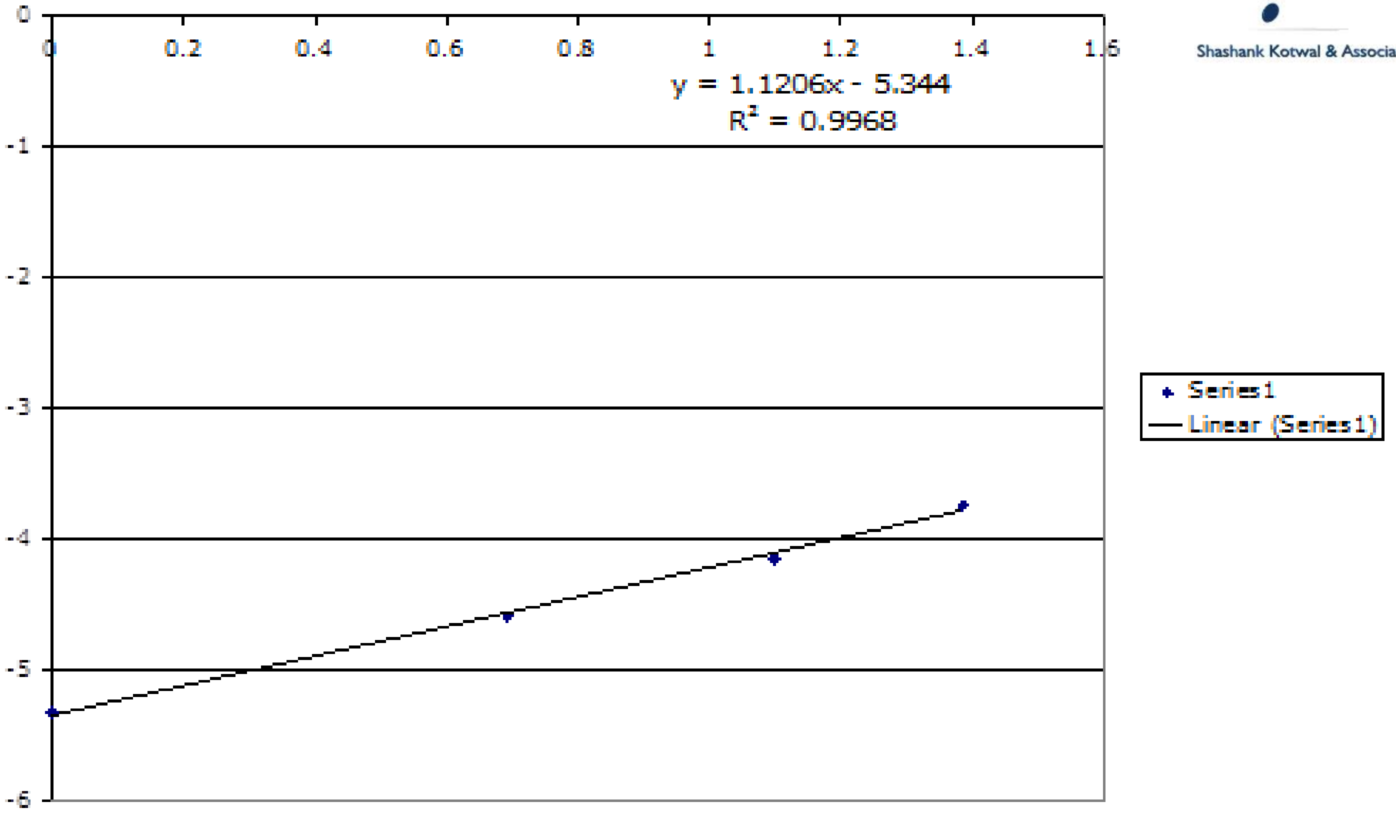
STEP

The data for the Weibull plot is arranged.

Time in Service (MIS)	% Failed (Median Ranks)
1	0.486%
2	1.01%
3	1.56%
4	2.33%

STEP

The Weibull plot is constructed using MS-Excel.



STEP

Extrapolate the regression equation to estimate the number of failures up to the desired time in service.

- The Weibull regression equation is:
- Y=1.1206x-5.344
- $$\ln\left(\ln\left(\frac{1}{1-F(t)}\right)\right) = 1.1206 * \ln(t) - 5.344$$
- it is needed to estimate the number of failures up to warranty period (12 MIS).
- Therefore, t=12 is substituted in the Weibull regression equation

$$\ln\left(\ln\left(\frac{1}{1-F(t)}\right)\right) = 1.1206 * \ln(12) - 5.344$$

$$\ln\left(\ln\left(\frac{1}{1-F(t)}\right)\right) = -2.55941$$

$$\frac{1}{1-F(t)} = \exp(\exp(-2.55941))$$

$$\frac{1}{1-F(t)} = 1.08042$$

$$\frac{1}{1.08042} = 1 - F(t)$$

$$F(t) = 1 - \frac{1}{1.08042}$$

- F(t) =. 0744 ~ 7.44%
- By the end of warranty (12 MIS), (7.44% of 10000) ~ 744 units, are estimated to fail.

HOW CAN WE SERVE YOU?

- Facilitation at Design stage
- Facilitation at Testing stage
- Warranty Analysis
- Training

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Want us to call?

Email: shashank@sskotwal.in

Appendix

APPENDIX

Table of 5% Ranks

Sample size (n) → Rank ↓	1	2	3	4	5	6	7	8	9	10
1	0.0500	0.0253	0.0170	0.0127	0.0102	0.0085	0.0074	0.0065	0.0057	0.0051
2		0.2236	0.1354	0.0976	0.0764	0.0629	0.0534	0.0468	0.0410	0.0368
3			0.3684	0.2486	0.1893	0.1532	0.1287	0.1111	0.0978	0.0873
4				0.4729	0.3426	0.2713	0.2253	0.1929	0.1688	0.1500
5					0.5493	0.4182	0.3413	0.2892	0.2514	0.2224
6						0.6070	0.4793	0.4003	0.3449	0.3035
7							0.6518	0.5293	0.4504	0.3934
8								0.6877	0.5709	0.4931
9									0.7169	0.6058
10										0.7411

Sample size (n) → Rank ↓	11	12	13	14	15	16	17	18	19	20
1	0.0047	0.0043	0.0040	0.0037	0.0034	0.0032	0.0030	0.0029	0.0028	0.0026
2	0.0333	0.0307	0.0281	0.0263	0.0245	0.0227	0.0216	0.0205	0.0194	0.0183
3	0.0800	0.0719	0.0665	0.0611	0.0574	0.0536	0.0499	0.0476	0.0452	0.0429
4	0.1363	0.1245	0.1127	0.1047	0.0967	0.0910	0.0854	0.0797	0.0761	0.0725
5	0.2007	0.1824	0.1671	0.1527	0.1424	0.1321	0.1247	0.1173	0.1099	0.1051
6	0.2713	0.2465	0.2255	0.2082	0.1909	0.1786	0.1664	0.1575	0.1485	0.1396
7	0.3498	0.3152	0.2883	0.2652	0.2459	0.2267	0.2128	0.1990	0.1887	0.1781
8	0.4356	0.3909	0.3548	0.3263	0.3016	0.2805	0.2601	0.2449	0.2298	0.2183
9	0.5299	0.4727	0.4274	0.3904	0.3608	0.3350	0.3131	0.2912	0.2749	0.2587
10	0.6356	0.5619	0.5054	0.4600	0.4226	0.3922	0.3542	0.3429	0.3201	0.3029
11	0.7616	0.6613	0.5899	0.5343	0.4893	0.4517	0.4208	0.3937	0.3703	0.3469
12		0.7791	0.6837	0.6146	0.5602	0.5156	0.4781	0.4460	0.4196	0.3957
13			0.7942	0.7033	0.6366	0.5834	0.5395	0.5022	0.4711	0.4434
14				0.8074	0.7206	0.6562	0.6044	0.5611	0.5242	0.4932
15					0.8190	0.7360	0.6738	0.6233	0.5809	0.5444
16						0.8274	0.7475	0.6871	0.6379	0.5964
17							0.8358	0.7589	0.7005	0.6525
18								0.8441	0.7704	0.7138
19									0.8525	0.7818
20										0.8609

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APPENDIX

Table of 95% Ranks

Sample size (n) → Rank ↓	1	2	3	4	5	6	7	8	9	10
1	0.9500	0.7764	0.6316	0.5271	0.4507	0.3930	0.3482	0.3123	0.2831	0.2589
2		0.9747	0.8646	0.7514	0.6574	0.5818	0.5207	0.4707	0.4291	0.3942
3			0.9830	0.9024	0.8107	0.7287	0.6587	0.5997	0.5496	0.5069
4				0.9873	0.9236	0.8468	0.7747	0.7108	0.6551	0.6076
5					0.9898	0.9371	0.8713	0.8071	0.7436	0.6965
6						0.9915	0.9466	0.8889	0.8312	0.7776
7							0.9926	0.9532	0.9032	0.8500
8								0.9935	0.9590	0.9127
9									0.9943	0.9632
10										0.9949

Sample size (n) → Rank ↓	11	12	13	14	15	16	17	18	19	20
1	0.2384	0.2209	0.2058	0.1926	0.1810	0.1726	0.1642	0.1559	0.1475	0.1391
2	0.3644	0.3387	0.3163	0.2967	0.2794	0.2640	0.2525	0.2411	0.2296	0.2182
3	0.4701	0.4381	0.4101	0.3854	0.3634	0.3438	0.3262	0.3129	0.2995	0.2862
4	0.5644	0.5273	0.4946	0.4653	0.4398	0.4166	0.3956	0.3767	0.3621	0.3475
5	0.6502	0.6091	0.5726	0.5400	0.5107	0.4844	0.4605	0.4389	0.4191	0.4036
6	0.7287	0.6848	0.6452	0.6096	0.5774	0.5483	0.5219	0.4978	0.4758	0.4556
7	0.7993	0.7535	0.7117	0.6737	0.6392	0.6078	0.5792	0.5340	0.5289	0.5068
8	0.8637	0.8176	0.7745	0.7348	0.6984	0.6650	0.6458	0.6063	0.5804	0.5666
9	0.9200	0.8755	0.8329	0.7918	0.7541	0.7195	0.6869	0.6571	0.6297	0.6043
10	0.9667	0.9281	0.8879	0.8473	0.8091	0.7733	0.7399	0.7088	0.6799	0.6531
11	0.9953	0.9693	0.9335	0.8953	0.8576	0.8214	0.7872	0.7551	0.7251	0.6971
12		0.9957	0.9719	0.9389	0.9033	0.8679	0.8336	0.8010	0.7702	0.7413
13			0.9960	0.9737	0.9426	0.9090	0.8753	0.8425	0.8113	0.7817
14				0.9963	0.9755	0.9464	0.9146	0.8827	0.8525	0.8215
15					0.9966	0.9773	0.9501	0.9203	0.8901	0.8604
16						0.9968	0.9784	0.9534	0.9239	0.8949
17							0.9970	0.9795	0.9548	0.9275
18								0.9971	0.9806	0.9571
19									0.9972	0.9817
20										0.9974

APPENDIX

- R90C90 Life
- Mathematical explanation for using confidence bound using MS-Excel

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]$$ the reliability function is

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

OR

$$t = \theta\left[\ln\left(\frac{1}{R}\right)\right]^{\frac{1}{\beta}}$$

OR

$$t_L = L\left[\ln\left(\frac{1}{R_{(t_L)}}\right)\right]^{\frac{1}{\beta}}$$

where, t_L = **R90C90** life, and, **L** = likelihood function of the unknown parameter θ

L is given by,

$$L = \left[\frac{2T}{\chi^2_{\alpha,v}}\right]^{\frac{1}{\beta}}$$

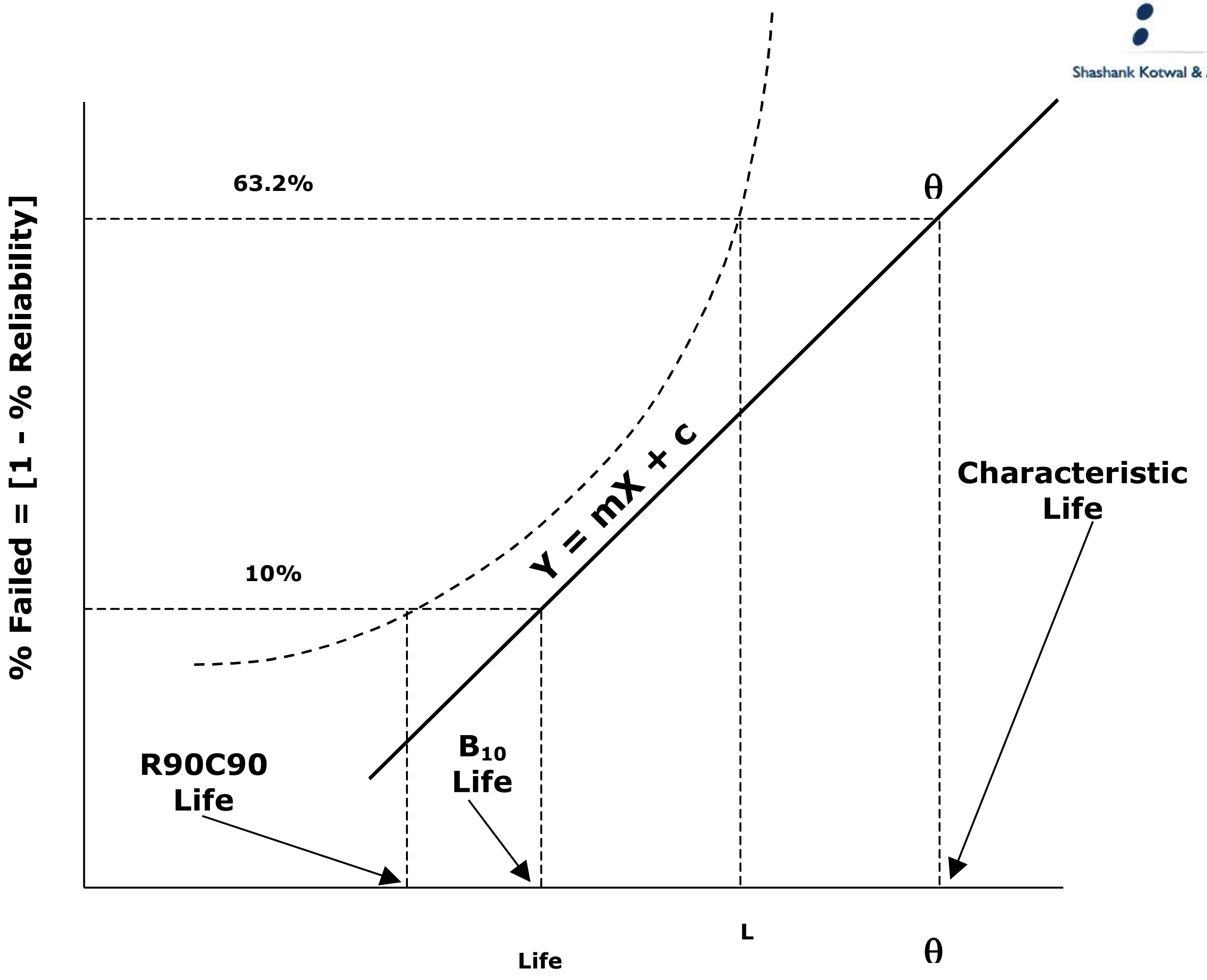
where,

$$T = \sum_{i=1}^{n}(t_i)^{\beta}$$

where t_i = cycles to failure of each of n samples tested.
And is the **100(1- α)th** percentile of chi-squared with ν degrees of freedom, where α = (1-%rank), which for C90 becomes, (1-0.9) = **0.1**

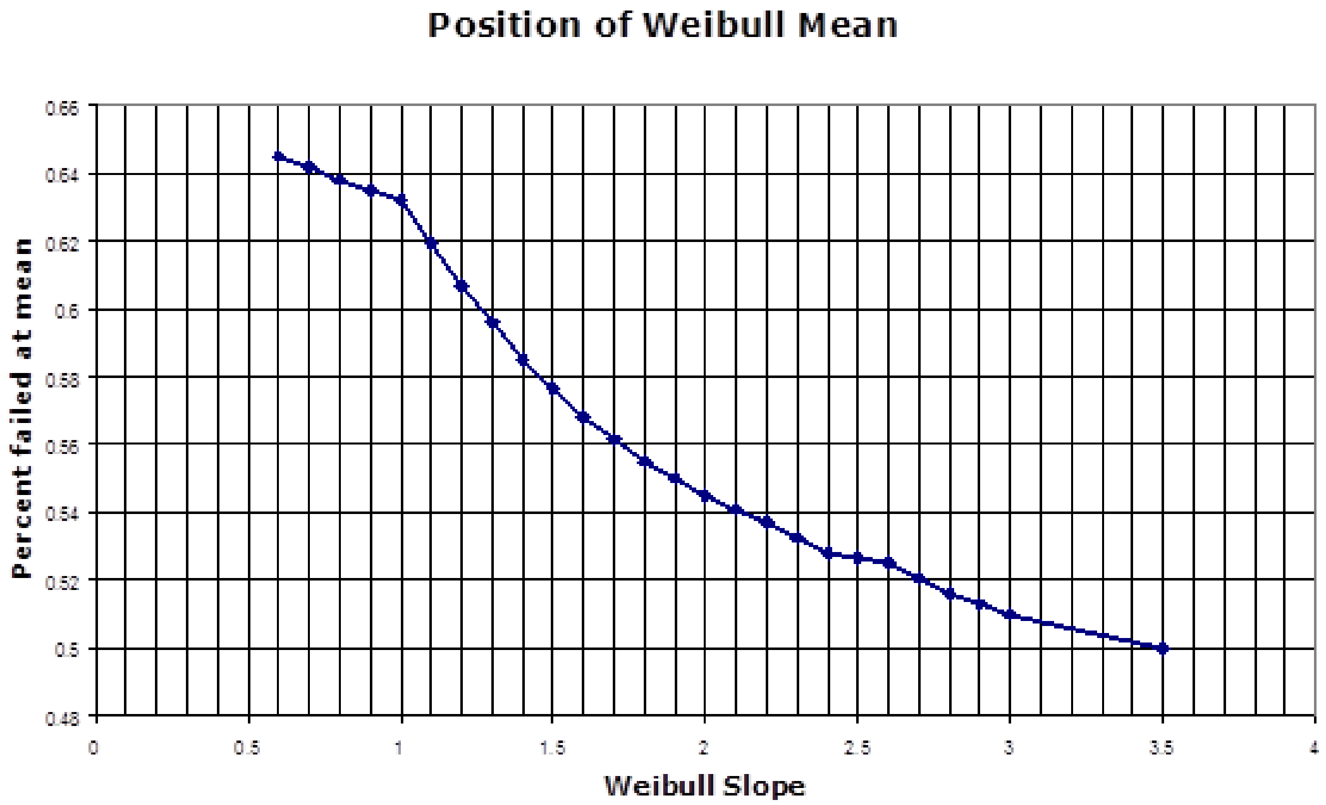
APPENDIX

- B10 compared with R90C90
- As is evident from the plot below, R90C90 gives a pessimistic estimate of the B10 life



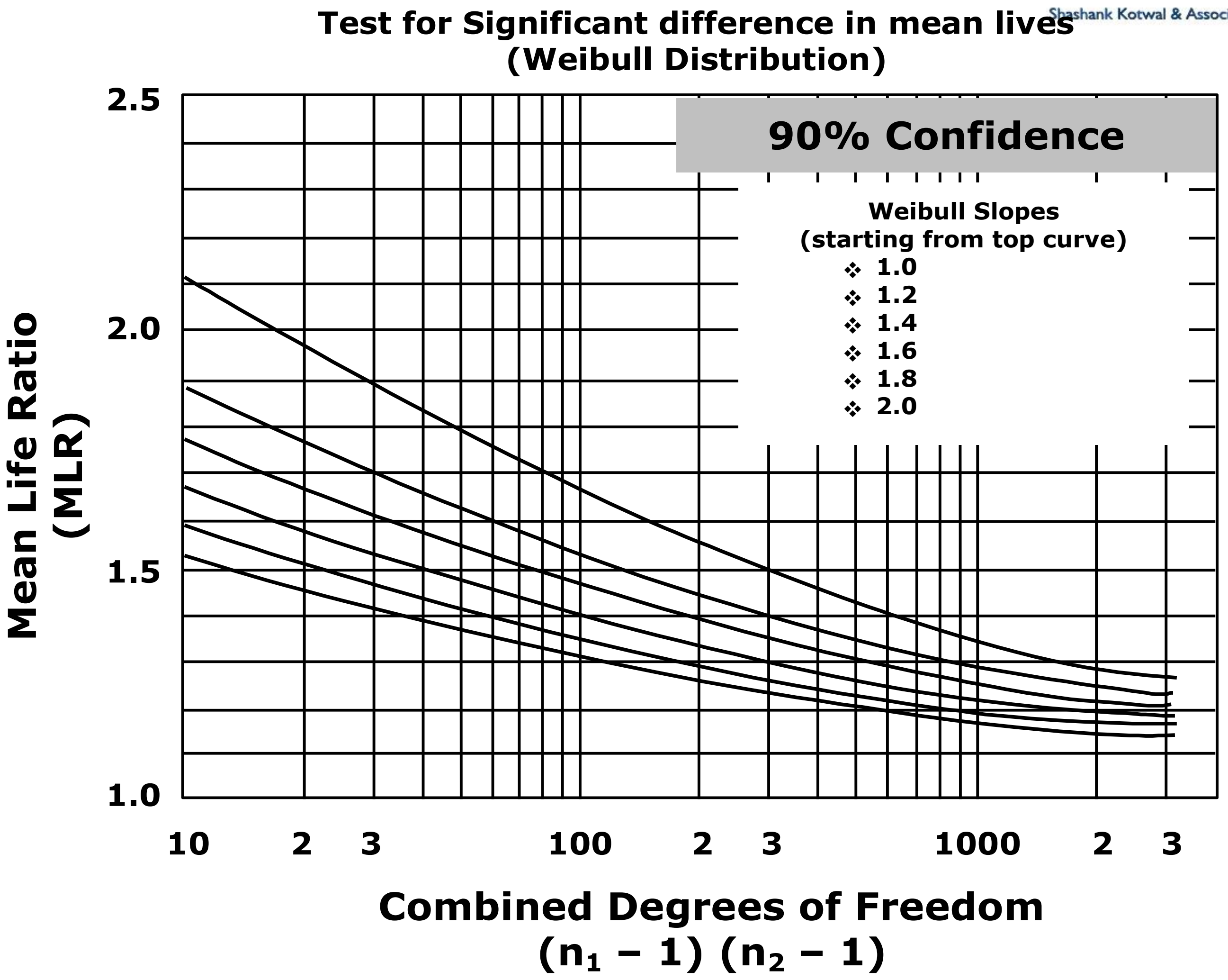
APPENDIX

Position of Weibull Mean



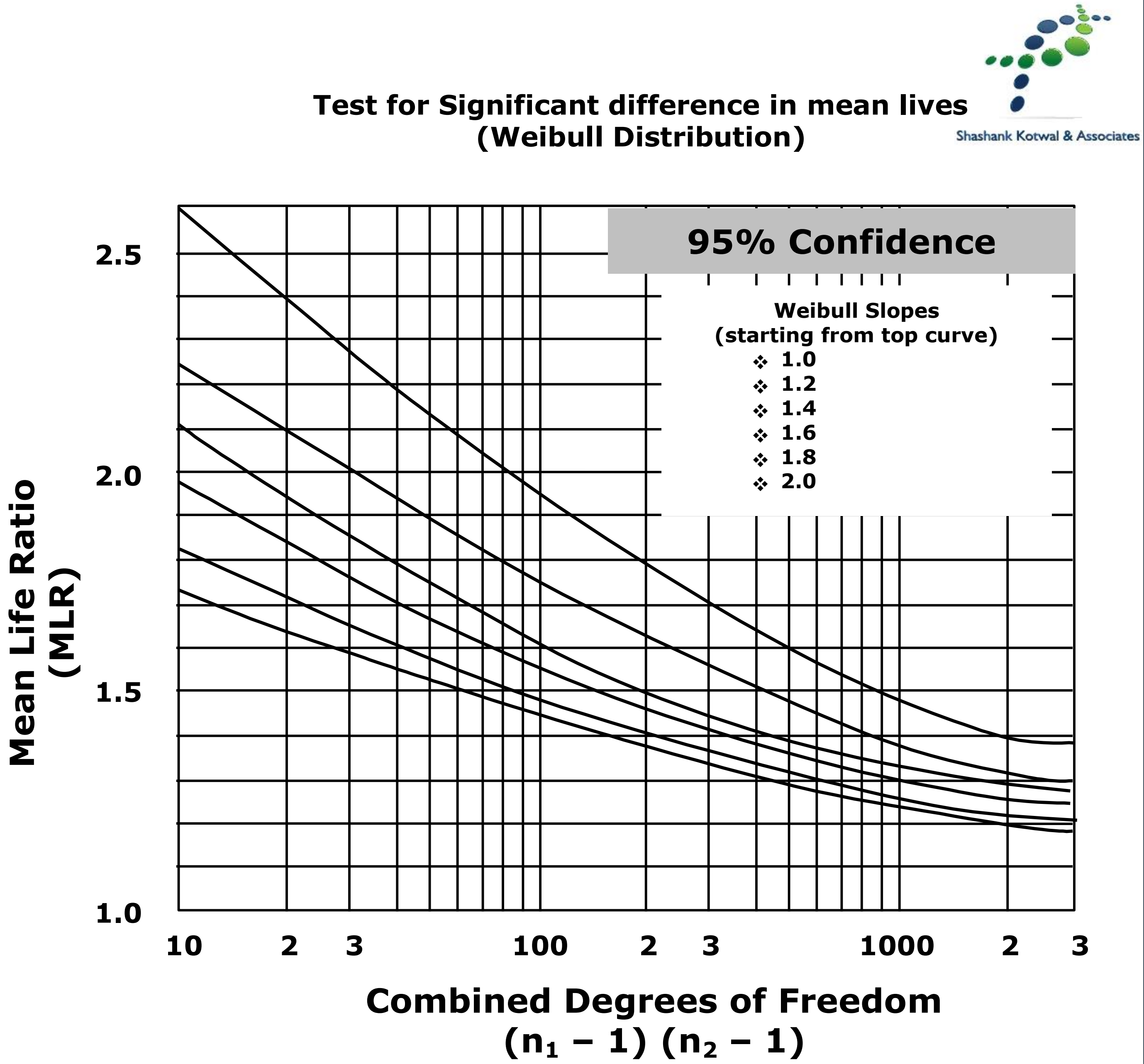
APPENDIX

Test for significant difference in mean lives @ 90% confidence



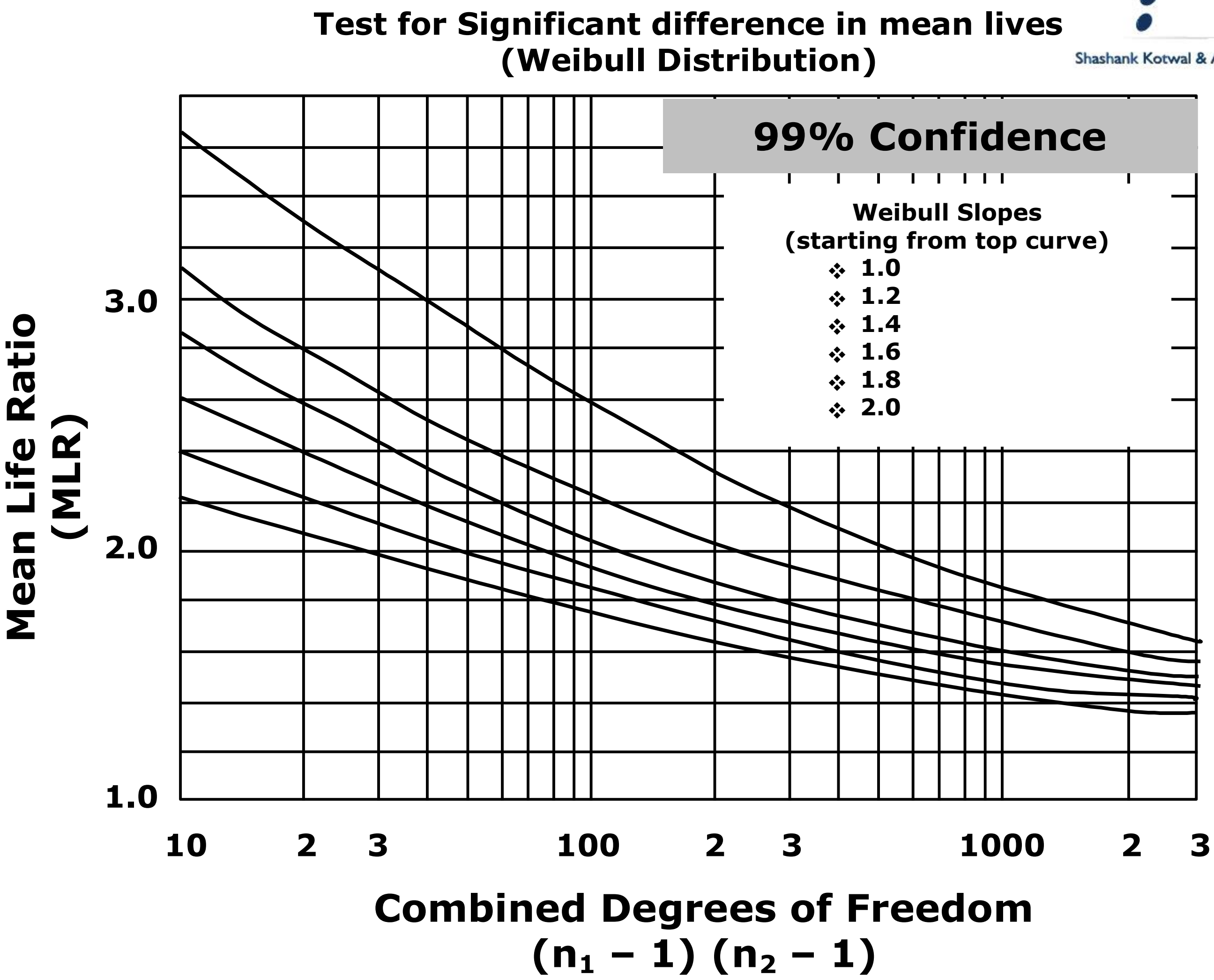
APPENDIX

Test for significant difference in mean lives @ 95% confidence



APPENDIX

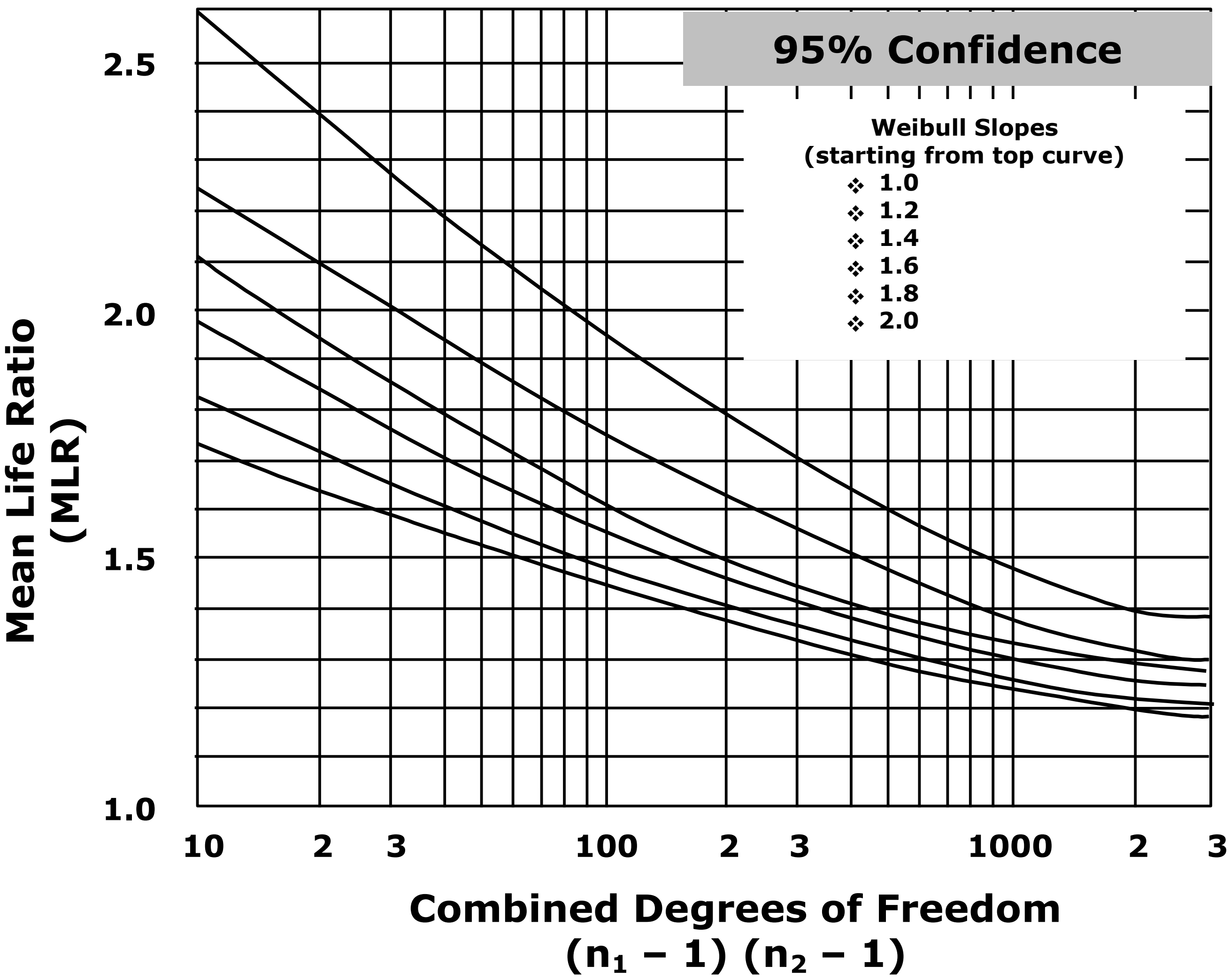
Test for significant difference in mean lives @ 99% confidence



APPENDIX

Test for significant difference in mean lives @ 95% confidence

Test for Significant difference in mean lives (Weibull Distribution)



Thank you